

Tony Cai is good at statistics , and the statistical inferences in high-dimension may be useful

[8.11]

- Main references

[1] Donoho, D.L. & Jin, J. (2004). Higher criticisms for detecting, sparse and heterogeneous mixtures. *Ann. Statist.*

Detection of homoscedastic Gaussian mixtures

[2] Cai, T.T., Jeng, J. & Jin, J. (2011) Optimal detection of heterogeneous and heteroscedastic mixtures *J. Roy. Statist. Soc.*

Detection of heterogeneous & heteroscedastic Gaussian mixtures

[3] Cai, T.T. & Sun, W. (2017). Optimal screening and discovery of sparse signals with applications to multistage high-throughput studies. *J. Roy. Statist. Soc. Ser. B.* 79.197-233

Screening & discovery of sparse signals

[4] Jeng, J. Cai, T.T. & Li, H. (2010) Optimal sparse segment identification with application in copy number variation analysis. *J. Amer. Statist. Assoc.* 105.1156-1166

Detection of segments-applications to CNV analysis

[5] Butucea, C. & Ingster, Yu. I. (2013) Detection of a sparse submatrix of a high-dimensional noisy matrix. *Bernoulli* 19. 2652-2688

Detection of submatrices

[6] Cai, T.T. & Wu, Y. (2014) Optimal detection for sparse mixture against a given null distribution. *IEEE Trans. Inf. Theory* 60

Detection of general (not necessarily Gaussian) mixtures

[7] Hall, P & Jin, J. (2008) Properties of Higher Criticism under long range dependence. *Ann. Statist.* 36

Detection of segments based on multiple sequences

[8] Hall, P & Jin, J. (2010) Innovated Higher Criticism for detecting sparse signals in correlated noise. *Ann. Statist.* 38

[9] Jeng, J., Cai, T.T., Li, H. (2013) Simultaneous discovery of rare and common segment variants. *Biometrika* 100

[8]-[9] Detection under dependency

[10] Cai, T.T., Jin, J. & Low, M.G. (2007) Estimation and confidence sets for sparse normal mixtures. *Ann. Statist.* 35

[11] Jin, J. & Cai, T.T. (2007) Estimating the null and the proportion of non-null effects in large-scale multiple comparisons. *J. Amer. Statist. Assoc.* 102

[12] Cai, T.T. & Jin, J. (2010) Optimal rates of convergences for estimating the null and proportion of non-null effects in large-scale multiple testing. *Ann. Statist.* 38

[10]-[12] Estimation of proportion (and null distribution)

- Theme

Talking about big data and big values, showed many applications of big data, especially about biology

【8.12】

- Theme

1. Questions: how can we infer whether a distribution is detectable or undetectable??
Tony give one picture showing the detectable status and undetectable status, keep an eye on lemmas (including proof)
假设检验中, 当概率足够小时拒绝 H_0 . 多小是小? ? detection boundary 给出了指示
2. Motivation for Higher Criticism (results from empirical process theory, see e.g. Shorack & Wellner(2009))
3. HC 方法统计推断流程: 计算 p 值; p 值标准化并排序; 进行 HC 统计, 当 HC 值过大时拒绝 H_0
4. Testing procedures: linear statistic & linear test; scan statistic and scan test; final test
5. 具体细节参见 Cai, T.T., & W. (2017) Optimal screening and discovery of sparse with applications to multistage high-throughput studies. J. Roy. Statist. Soc. Ser. B.79

- Other References: Mentioned during class

- [1] Detection: Ingster(1999) and Donoho & Jin (2004)
- [2] Estimating the fraction p: Meinshausen & Rice (2006), Cai, Jin and Low(2006), Jin & Cai(2007)
- [3] Detection boundary(Ingster, 1999; Donoho & Jin,2004)
- [4] Hellinger and total variation metrics (they are different)
Remarks: lemma of Inequalities relating Hellinger and total variation metrics
- [5] Cai, Jeng & Jin (JRSSB, 2011) consider the detection problem in the heteroscedastic case
Sparse case and dense case

【8.13】

- Theme

1. Submatrix localization, see it in Cai, T.T., Liang, T. & Rakhlin, A.(2017) Computational and statistical boundaries for submatrix localization in a large noisy matrix. Ann. Statist. 45
2. Robust sparse segment detection & identification: consider the noise distribution is not normal and unknown, see it in Cai, Jeng & Li(JRSSB,2012) considerer robust detection and identification when noise distribution is unknown and hard to be estimated.

3. Some interesting sparse signals detection problems:
 - Detecting sparse Gaussian mixtures;
 - Detecting of general (not necessarily Gaussian) mixtures
 - Detecting submatrices
 - Detecting Sparse segments in Gaussian noise
 - Robust detection of sparse segments
 - Detecting sparse segments with multiple samples
 - Detecting principal components (PCA algorithm)

 4. Testing a small number of hypothesis(Bonferroni method), testing a large number of hypotheses
 5. FDR control(BH step-up procedure; proof of BH procedure);
 - Improving BH procedure(Meinshausen & Rice(2006), Cai, Jin & Low(2007). Jin & Cai(2007). Cai & Jin(2010)
 - Other FDR procedure:Adaptive procedure(Benjamini and Hochberg,2000);Plug in procedure(Genovese and Wasserman,2004)
 - FDR control based on p value, then give the proof of why we use p value as the evidence against H_0
 6. Revisit Robbins(1951),see it in Robbins, H. (1951), Asymptotically subminimax solutions of compound statistical decision problems. Proc. 2nd Bereley Symp. Math. Statist. Probab. UC Berkeley
 7. Data-Driven procedure: see it in <http://www-stat.wharton.upenn.edu/~tcai/paper/html/FDR.html>
 8. Estimate p, f_0 and f with the desired properties. For estimating p and f_0 ,see,Jin & Cai(2007) and Cai & Jin(2010);
 - When all the assumptions are correct, the p values and z values follow their theoretical null distribution under the null
 - The theoretical nulls could be quite different from the empirical nulls in applications. See Efron(2004) , Jin & Cai(2007), and Cai & Jin(2010)
 - Uniformly consistent estimates of p and f_0 are given in Jin & Cai(2007) and optimal rates are estimated in Cai & Jin(2010)
 - Estimating f is a classical problem in nonparametric function estimation. See, e.g. Silverman(1986)
- Other references
- [1] Efron. B.(2004) Large scale simultaneous hypothesis testing: The choice of a null hypothesis. J. Amer. Statist. Assoc 99,96
 - [2] Efron, B. (2008) . Microarrays, empirical Bayes and the two groups model . Statist. Sci. 23,1-22
 - [3] Benjamini, Y. & Yekutieli, D. (2001). The control of false discovery rate in multiple testing under dependency. Ann. Statist. 29

- [4] Copas, J.(1974). On symmetric compound decision rules for dichotomies. *Ann. Statist.* 2,199-204
- [5] Jin, J. & Cai, T.T. (2007). Estimating the null and the proportion of non-null effects in large-scale multiple comparisons. *J. Amer. Statist. Assoc.*102
- [6] Cai, T.T. & Jin, J.(2010) Optimal rates of convergence for estimating the null and proportion of non-null effects in large-scale multiple testing. *Ann. Statist.* 38
- [7] Cai, T.T.(2017) Global testing and large-scale multiple testing for high-dimensional covariance structures. *Annu. Rev. Stat. Appl.* 4
- [8] Benjamini & Hochberg(1995,JRSSB): FDR 上界

【8.14】

- Theme
 1. Compressed sensing: Developing efficient algorithm to recover the unknown signal; Finding condition on A under which it is possible to recover beta accurately; constructing “good” sensing matrices A such that it is possible to recover sparse signals beta using an efficient algorithm
 2. Sparse signal recovery under MIP, see it in Cai, T.T., Wang, L. & Xu. G.(2010) Stable recovery of sparse signals and an oracle inequality. *IEEE, Transactions on information Theory* 56; Cai, T.T. & Zhang, A.(2013) Compressed sensing and affine rank minimization under restricted isometry, *IEEE Trans. Signal Process* 61; Cai, T.T. & Zhang, A. (2013). Sparse representation of a polytope and recovery of sparse signals and low-rank matrices. *IEEE Trans. Inf. Theory* 60
 3. Understanding the relationships among the l_0 (sparsity), l_1 (objective function), l_2 (loss) norms.
 4. Compressed sensing under RIP : different conditions on delta and theta have been used in the literature, see these in Candes and Tao(2005); Candes, Romberg and Tao(2006); Cades and Tao(2007); Candes(2008); Cai, Xu and Zhang(2009); Cai, Wang and Xu (2010); Cai and Zhang(2013a); Cai and Zhang(2013b)
 5. A variety of sufficient conditions on the RIC delta for the exact/stable recovery of k-sparse signals have been introduced in the literature. See these in Candes(2008); C., Wang and Xu(2012a); C., Wang and Xu(2010c)
 6. Coherence of random matrices: limiting laws, phase transition ,& application
 7. Problems about economics: see these in Andrews(*Econometrica*,1991) , Ligeralde and Brown (*International Economic Review*,1995)
- References
 - [1] Cai, T.T., Wang, L. & Xu, G. (2010) Stable recovery of sparse signals and an oracle inequality. *IEEE Trans. Inf. Theory* 56
 - [2] Cai, T.T., & Zhang, A. (2013) Compressed sensing and affine rank minimization under restricted isometry. *IEEE Trans. Signal Process*, 61

- [3] Cai, T.T. & Zhang, A.(2014) Sparse representation of a polytope and recovery of sparse signals and low-rank matrices. IEEE Trans. Inf. Theory 60
- [4] Cai, T.T. & Jiang, T.(2011) Limiting laws of coherence of random matrices with application to testing covariance structure and construction of compressed sensing matrices. Ann. Statist. 39
- [5] Baraniuk, R., Davenport, M., DeVore, R. & Wakin, M.(2008) A simple proof of the restricted isometry property for random matrices. Constr. Approx. 28
- [6] Bickel, P. J., Ritov, Y. & Tsybakov, A. B. (2009) Simultaneous Analysis of Lasso and Dantzing Selector. Ann. Statist.
- [7] Cai, T.T. & Zhang A.(2013) Sharp RIP bound for sparse signal and low-rank matrix discovery. Appl. Comput.
- [8] Cai, T.T., Wang, L. & Xu, G.(2010). Shifting Inequality and recovery of sparse signals. IEEE Trans. Signal Process. 58
- [9] Candes, E.J. & Tao, T. (2007) The Dantzing selector: statistical estimation when p is much larger than n (with discussion). Ann. Statist. 35
- [10] DasGupta, S. & Gupta, A.(1999) An elementary proof of the Johnson-Lindenstrauss lemma
- [11] Donoho, D.L. (2006). Compressed sensing. IEEE Trans. Inf. Theory 52
- [12] Donoho, D.L., Elad, M. & Temlyakov, V.N. (2006) Stable recovery of sparse overcomplete representations in the presence of noise. IEEE Trans. Inf. Theory 52
- [13] Johnson, W. B. & Lindenstrauss, J. (1984) Extension of Lipschitz mappings into a Hilbert space. Contemp. Math. 26
- [14] Cades & Tao(2005,2007). Bickel, Ritov & Tsybakov(2008), Cai, Wang, & Xu(2010a,b), Cai and Zhang (2013a, b. 2014)
- [15] Donoho & Huo(2001), Fuchs(2004,2005), Donoho, Elad & Temlyakov(2006) , Tropp(2004,2006), Cai, Wang & Xu(2010a, b) ,Cai and Zhang(2013a,b,2014)
- [16] Cai, T.T., & Jiang. T. (2012). Phase transition in limiting distributions of coherence of high-dimensional random matrices, J. Multivariate Anal

Remarks:

Others you may interested about statistics:

Testing grouped hypothesis

Large-scale multiple testing with covariates

Integration large-scale data analysis & statistical inference

Statistical inference for high-dimensional linear regression

Inference for large matrices

Supervised learning

Unsupervised learning